Principles Behind Points Awarded in Three-Way Chess. (DRAFT)

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Points are awarded to players based on the following rules:

I. Standard chess point-awards still apply: a player earns 1 point if (s)he defeats an opponent and 1/2 point if (s)he ties a (single) opponent.

II. A new award is added: a player (also) earns 1/2 point if (s)he survives undefeated in play against two simultaneous opponents.*

III. Final game scores are sums: Each player's final score is the *sum* of points earned according to both principles (and in both stages of the game, if it is Second Mate).

Motivation for additional half-point award and rationale for award sizes and use of sums

When deciding on a scoring scheme for Three-way Chess, the obvious initial idea is to simply adopt the rule used in standard chess: award one point to "the winner" (i.e., to the mating player in a First Mate game, or to the last surviving player in a Second Mate game) or, if there is a tie, to divide the award and give 1/2 (or 1/3) point to each player. However, this solution has two serious shortcomings.

The first shortcoming is internal: it does not adequately reflect relative values of player outcomes within a single game. This is because three-way struggle has an intermediate level of success which is not possible when there are just two players..

The second shortcoming is external: it does not adequately reflect the relative values of player outcomes in three-way vs. two-way games. This is because a three-player game intrinsically involves more struggle and accomplishment than a two-player game.

A three-way struggle incorporates more player confrontations (in one sense, three simultaneous pairwise contests). However, the outcomes for these are not fully independent, so (as explained below) a three-player game actually involves independent between-player contests equivalent to two standard two-way chess games. In addition, a Second Mate game incorporates a further pairwise contest in Stage 2, and so in inter-player contests is equivalent to three standard two-way chess games.†

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* This does not apply to a three-way tie; in that case, each player's total score is 2/3 point.

† If one considers the "breadth", "depth", and the "dimensions of thinking" involved, it can be argued that victory in Three-way Chess should be given even higher point values relative to standard two-way chess than would be indicated by our simple computations based on the number of player-confrontations and outcome degrees of freedom that are involved. This is discussed in a separate note: "Comparison of Breadth, Depth, and Kinds of Thinking required by Three-way vs. Two-way Chess."

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Because of these characteristics, player outcomes in Three-way Chess are evaluated in terms of the rules listed above, and each player is awarded an often multipoint cumulative game score. The relative merits of different schemes, and the choice of this scoring scheme, was based on the following four important (and somewhat interdependent) objectives:

(1) "to be fair":
The relative sizes of the three players" scores should reflect our intuitions about their relative level of outcome success.

(2) "to be consistent":
A given number of points should represent a given total amount of competitive accomplishment (game-outcome successes) regardless of the type, or stage, or number of chess games in which the points were earned. It should represent enough fully successful opponent-confrontations plus enough part-scores for partly successful ones to sum to the given point total. Chess points should all measure accomplishment on the same scale, whether earned in Stage 1 or 2 of Three-way Chess or in standard two-way chess.
(3) "to be strategically neutral":
The point values awarded for different kinds of game outcomes (victory, survival, or ties, in stage one or two) should not bias players toward aggressive or defensive strategies.

(4) "to have a good quantitative basis":
The relative sizes of awards for each outcome should have a reasonable quantitative rationale and, if possible, approximate ratio-scale measurement of the outcome success.

Explanation of the four objectives and how the proposed rules help meet them.

1. **"to be fair".**

Clearly, being defeated is not as good as surviving, and this is not as good as surviving *and* defeating an opponent. It follows that, when there are three players, a simple win-vs.-lose, 1 vs. 0 scoring is inadequate to fairly reflect the relative level of *each* player accomplishment.

The question then becomes, how much better is surviving than not, and how much better is defeating an opponent than merely surviving? If we take, as a basis for our scale, the two-way chess convention that 1 point is awarded for victory and .5 for a tie, then the only size question to resolve is the amount to be given to the two surviving players for not being defeated in a three-way match. There are several considerations that lead to choosing a of value .5.

(a) An amount of "only" .5 for survival seems *intuitively fair* (to me, at least), because managing to survive in three-way play seems to be clearly less of an accomplishment than defeating an opponent. The simplest proportion, ½, seems not unreasonable.

(b) As a consequence, a player who survives against two opponents *and* defeats one of them ends up with 1.5 points. This also seems intuitively fair since it is a bigger accomplishment to defeat an opponent in a three-player struggle, where one has to simultaneously defend against another opponent. And there is also a competitive accomplishment. If you are player A and you defeat B, you have not only accomplished a victory over B, you have implicitly accomplished a (smaller) victory over C in that you have defeated B before C could.

(c) On the other hand, an alternate way to look at the .5 awarded the two surviving players is that it reflects their implicit or technical tie. It does seem reasonable to ask how we should interpret (and award points for) the outcome status of the two undefeated players relative to one another. If the question of who defeated an opponent first is "already taken care of" by the award of 1 point to the victorious player, this leaves the simple fact that we have two survivors, neither of whom managed to defeat the other. Thus they have (in this narrow respect) tied one another. Giving an additional .5 to the two survivors of a three-player match might simply be a recognition of this fact and an application to them of the standard award for a tie.[i]

(d) A strong general argument for the choice of .5 follows from considerations of the degrees of freedom of the outcomes of a three way match. This is explained below under discussion of quantitative justifications.

2. "to be consistent".

It is very desirable that all chess points should be interchangeable (i.e., be "currencies of equivalent value"). In this way, they can be easily interpreted and they can be summed across stages and/or across games of different kinds (First Mate, Second Mate, and classic two-way chess), to compute total point scores for a given player, a team of players in a given tournament, or for one or more players over some particular period.

If it is agreed that one defeat of an opponent is approximately equal to two ties and this approximately equals two successfully non-defeated outcomes in three-way play, then the individual amounts awarded in 3WC seem appropriate. By also adopting the summation principle (III at the start of this note), the objective of maintaining the meaning of overall player point scores seems also to follow. (The consistency with two-way chess is further supported if we take the interpretation of the half point awarded for survival as another name for a half-point awarded for two tieing players.)

3. "to be strategically neutral" .

In a Second Mate game, a one-point scoring rule in which "the final winner takes all", would encourage a strategy of playing very defensively in Stage 1, the three-player stage, in order to preserve one pieces and position and survive to Stage 2 in the strongest possible shape (i.e., "let you and him fight"). In contrast, the proposed scoring rules do not reward one for "laying low" in Stage 1. Examination of the table of points earned by different outcomes reveals that someone who merely survives the first stage can, at best, tie his Stage 2 opponent in terms of overall score. And things could come out worse. A tie at Stage 2 results in loosing overall by 1 point, and losing puts you down by a full 2 points.

In a First Mate game, conservative initial play is still a feasible strategy. But since whoever defeats anyone will win the game, you could lose to an aggressive player who defeats the third player before you can prevent it. Timing of one switch to offense becomes quite critical. One must also insightfully forsee all the implicit threats hidden in the position of pieces of one or both of your opponents.

4. "to have a good quantitative basis" .

The proposed scoring scheme appears to be the only one (that I have come up with so far) that possesses the following appealing quantitative property: the total points awarded in any game correspond to the game "degrees of freedom" (the number of independent aspects of the game possible outcomes). To see this, note that (i) A total of one point is awarded for outcome success

in a two-player confrontation. This is natural because such a game has only one degree of freedom--once one player status is specified, the remaining player status is determined. (ii) In a single three-player engagement, it takes information on the outcome status of two players to determine the status of the third player. It therefore is logical that, to reflect the number of independent aspects of outcome success in such a game, two points in total need to be awarded, though they can be distributed in different ways among the three players. (iii) The outcomes of a full Second Mate game (ignoring differences in outcome patterns that can be removed by permutation of player labels) have three degrees of freedom. It requires specification of the outcome of two players in Stage 1 and one player in Stage 2 to determine the remaining two outcome results. Thus, the total points to be awarded for the overall game, at the end of both stages, should sum to 3. (This also follows from (i) and (ii) so long as the score summation principle is followed.) Looking at the possible game profiles in the table of all possible outcomes (Appendix 1) shows how the totals of a First Mate game always sum to 2 and the totals of a Second Mate game always sum to 3. (A special case occurs if two players manage to checkmate the third simultaneously and independently. In this case there is actually an extra degree of freedom for the game, and the total score awarded in the table reflects this.)

Consideration of degrees of freedom also strengthens the case for awarding 1-point for defeating an opponent vs. ½ for survival against two opponents. Survival is always true of two players out of three, and hence the point (representing a single degree of freedom) must be divided between the two players involved. In contrast, the event of defeating an opponent typically happens only to one player. Its independent degree-of-freedom can be assigned specifically to that player. (When two players decide to jointly checkmate someone, the checkmate still represents only 1 degree of freedom, which thus must be divided between the cooperating players.

Finally, it was consideration of degrees of freedom in a standard Stage 1 game that provided the basis for awarding precisely 2/3 points to each player in the case of a three-way tie. Interestingly, this amount also seems desirable and sensible on independent grounds. Securing a tie is arguably a bit *more* of an accomplishment than simply surviving (which gets ½ point), yet it also seems a bit *less* of an accomplishment that defeating an opponent (which gets 1 point). The value of 2/3 fits appropriately in between (and, perhaps, is appropriately is closer to survival than victory--but here we are probably getting into overly precise evaluations of fuzzy concepts).

Appendix 1: The reverse victory scenario

Let consider in detail the intuitive appropriateness of the awards given in the following particular outcome of a Second Mate game:

	Points awarded to		
	A	B	C
A beats B then C beats A	1.5	0	1.5

At first glance it might seem unclear that A and C should get equal scores. After all, C ends up beating A. However, C only beat A *after* A had fought and won a battle with B, presumably at some cost to A in pieces and position relative to C. Thus, C eventual victory in Stage 2 cannot be taken as an argument that he should emerge from the game as a whole with a better score than A. Also, we should keep in mind that A and C do stand equal in terms of overall accomplishment—each defeated one opponent. We must also keep in mind that it just this

scoring result that helps ensure the strategic neutrality of the overall scoring scheme (as noted earlier).

Appendix2 : Comparing these rules to two "natural" and plausible alternatives.

There are a couple of interesting and initially plausible alternative choices of point award sizes for different outcomes that, for completeness and further insight, need to be considered and compared with the one that has been adopted. However, it seems to me that they both turn out to have flaws in terms of one or more of the 4 objectives listed earlier.

Awarding 1, .5, and 0. The first alternative is to award points in a three-player contest (i.e., Stage 1) as follows: the winner gets 1 point, the loser gets 0 and the surviving third player gets .5 points. This is simple and is suggested by analogy with classic scoring, where the winner also gets 1 point. One interpretation of this scoring could be that the additional score increment awarded for victory is the same size as the one award for survival, namely .5 points. The arguments against this proposal are as follows: (a) Isn"t victory a stronger accomplishment than survival (hence deserving a larger increment)? (b) Isn"t victory in three-way play a greater accomplishment than victory in two-way? (As noted earlier, shouldn"t defeating an opponent while also defending against a second opponent in a three-person contest be worth more, in total Stage 1 score, than a victory against a single opponent in a two-person contest?) If so, 1 point for "the winner" of a three-person contest is not enough. (c) What happens in the case of a tie? Securing a tie in a three-person contest should be worth more than simply surviving in a three person contest. (s) And what is awarded to A and C when they jointly checkmate B? (c) This approach also lacks the rationale and support based on degrees of freedom, as explained above, and, perhaps most clearly and quite importantly, (d) it does not make the awarded points as comparable to those earned in two-way chess (in terms of successful and half-successful outcomes attained).

Awarding 2, 1, and 0. The second "natural" alternative is to award 1 point for survival and 1 further point for defeating an opponent. Then the typical outcome scores would be 2 0 1. The weakness of this proposal, it seems to me, is that (a) survival would be overrated, since it would be made as valuable as victory, yet by the nature of the game there are always *two* surviving players; (b) again, points could not be as clearly justified in terms of fully successful and half successful opponent confrontation outcomes. Comparability with points earned in two-way chess would be undermined; (c) degrees of freedom would not be preserved or recognized.

Endnote:

[i] Of course, this pairwise evaluation then raises the question of what one should consider the status of the losing player (B) with respect to the surviving player that did not defeat him (C)? To me, this is a difficult question which in the end should probably be considered indeterminate or undefined at this level of analysis. For example, the fact that A defeated B rather than defeating C might be taken to suggest that B is weaker than C. However, it conveys as much or more information about the relative skill exhibited by A and C, who would both succeed by defeating B , as it does about the relative skill of B and C. To some extent, A"s defeat of B precludes obtaining information about the status of B and C.

Points Awarded to Players in All Possible Outcomes of Three-Way Chess

Point Rules:

Defeat an opponent = 1
Tie an opponent = 1/2
Outlive an opponent = ½

(Points earned are additive. Special cases: Jointly defeat opponent =1/2; Three-way draw = 2/3)

Case	Outcome Description	Points Awarded in *First Mate* game or in Stage 1 of 2			Points Awarded in Stage 2 of *Second Mate* game			Total score in *Second Mate* game		
		X	Y	Z	X	Y	Z	X	Y	Z
1	X defeats Y (X & Z survive)	1.5	0	.5						
	If then X defeats Z				1		0	2.5	0	.5
2	X defeats Y (X & Z survive)	1.5	0	.5						
	If then Z defeats X				0		1	1.5	0	1.5
3	X defeats Y (X & Z survive)	1.5	0	.5						
	If then X and Z tie				.5		.5	2.0	0	1.0
4	X, Y and Z agree to 3-way draw	2/3	2/3	2/3						
5	X and Z agree to jointly checkmate Y	1.0	0	1.0						
	If then X beats Z				1		0	2	0	1
6	X and Z agree to jointly checkmate Y	1.0	0	1.0						
	If then X and Z tie				.5		.5	1.5	0	1.5
7	X and Z both independently checkmate Y (very rare)	1.5	0	1.5						
	If then X beats Z				1		0	2.5	0	1.5
8	X and Z both independently checkmate Y (very rare)	1.5	0	1.5						
	If then X and Z tie				.5		.5	2.0	0	2.0

(Note: all other outcomes can be obtained by permuting labels assigned to players.)